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The updated ionospheric delay model to improve the performance of GPS single-frequency receivers

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Abstract. A regional updating technique is suggested for the global ionospheric delay model (*Klobuchar*'s [1987] model). The updating procedure uses series of values of the ionospheric delay (ID) which are reconstructed from distance measurements at two Global Positioning System working frequencies. We consider the situation where Klobuchar's model is updated from measurements from a single ground-based reference station. We exemplify the use of a regional updating of the ID model under undisturbed ionospheric conditions, as well as under solar eclipse and magnetic storm conditions. In all cases presented, using the proposed technique the standard deviation between predicted and measured ID values was reduced by a factor of 1.2–2.8 as compared with Klobuchar's original model used under the same conditions. A tentative estimate is made of the variation in the ionospheric rms error in determining the user's location (UPL) through the use of the regional updating technique. For the examples considered in this study, on applying the regional updating of Klobuchar's model, the ionospheric rms error in determining the UPL was found to be lower by a factor of 1.3–2.6 than that obtained by using the same model without updating.

1. Introduction

The satellite radio navigation system Global Positioning System (GPS) provides the user's accurate positioning in the horizontal plane using the Coarse/Acquisition (C/A) code, designated also as the Standard Positioning Service (SPS), of up to 100 m [*Hofmann-Wellenhof et al.*, 1992]. In this case, the main source of errors in determining the coordinates is the delay caused by the propagation of the radio signal from the navigation satellite (NS) in the ionosphere, along with errors of ephemeris and frequency-time support, multipathing, and tropospheric refraction. By measuring simultaneously the delay of radio signals emitted at two working frequencies of the GPS, it is possible to determine and eliminate the ionospheric error in positioning. However, the cost of double-frequency GPS receivers is prohibitive to the user on a mass scale. GPS-user receiving equipment (UE) employing a single working frequency of the system and functioning with the C/A code is used more extensively because of its relatively low cost, simplicity, and reliability.

For such receivers the accuracy in positioning can be improved through the use of a differential mode of GPS operation. In this approach one or several highly accurate double-frequency receivers are placed at known surveyed locations to monitor the received GPS navigational radio signals. At these reference stations the slowly varying components of the measurement error of the range to the visible NS are estimated, and corresponding range corrections are generated for each of them. These corrections are made available to all users of the GPS differ-

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ential system via the various data transmission lines. Typical corrections in the GPS differential system include three components of the ephemeris error, clock updating data, as well as the estimated ionospheric delay (ΔT) over a given region.

In so-called wide-area GPS differential systems with the size of a working area of about several thousand kilometers, the values of updating corrections are determined from a combined processing of GPS NS signals received by a large number of reference stations [*El-Arini et al.*, 1994]. At the same time, a widely accepted class of GPS differential systems are local differential systems (LDS) incorporating a single reference station, control equipment, and data transmission facilities. The operating range of such systems is 50-100 km, and they are used to support navigation in narrow passages of water such as straits and in water areas of ports and harbors, International Civil Aviation Organization (ICAO) final approaches of aircraft, as well as in supporting geodetic, land survey, and other kinds of special-purpose operations.

Such systems are more simple in design compared with wide-area differetial systems, but the accuracy of the corrections generated in them depends critically on the distance between the user and the reference station. In particular, this is especially true in regard to the ionospheric correction, which is expected to be the same throughout the operating range of the LDS and equal to the ionospheric delay from the NS to the reference station. The accuracy of such a correction of the ionospheric error decreases with distance from the reference station, especially under disturbed ionospheric conditions.

To reduce the ionospheric error of positioning of the LDS user, we propose that the ionospheric delay model by Klobuchar [1987] should be used, because its accuracy can be improved from a GPS signal monitoring at the reference station. Currently, Klobuchar's model is used in C/A code GPS receivers and provides a 50% reduction in ionospheric error in positioning under quiet geomagnetic conditions at middle latitudes. The accuracy of this model is lower under disturbed geomagnetic conditions in high-latitude and equatorial regions.

The use of Klobuchar's model, updated from current measurements, to determine the ionospheric corrections offers some advantages over conventional techniques for predicting total electron content (TEC) when determining the location using the LDS. Thus the algorithm used to produce global ionospheric maps [*Wilson et al.*, 1995] permits only the spatial TEC distribution to be reconstructed. Improvement in time resolution can be achieved by increasing the number of stations whose measurements are used in calculations. The algorithms for determining the ionospheric delay which were suggested for a Wide-Area Augmentation System (WAAS) are also based on ionospheric monitoring data from the network of reference stations [*El-Arini et al.*, 1994].

For the LDS based on a single reference station, it is possible to construct the space-time field of ionospheric-delay values over a given region using a TEC model dependent both on spatial coordinates and on time. The use of Klobuchar's model for this purpose significantly reduces the requirements for data transmission lines, via which it is necessary, in this case, to transmit only a refined set of coefficients (model parameters). Furthermore, this model forms a part of the software of existing GPS-receivers.

The aim of this paper is to suggest a technique for improving the accuracy of Klobuchar's model based on reconciling its values with the ΔT value inferred from measurements of navigation signal parameters at the reference station (RS) using double-frequency GPS receivers.

2. Technique for Updating the Ionospheric Delay Model

To update Klobuchar's model, we used series of measurements of the group and phase paths of NS signals to reconstruct the corresponding values of the group and phase ionospheric delay. Parameters of the model being updated were chosen in such a way that a minimum discordance between the model and measured ΔT values has been achieved. The method of determining the ΔT at frequency f_1 using measurements of the group and phase paths of NS radio signals at the GPS working frequencies f_1 and f_2 is well known [*Jorgenson*, 1978]:

$$\Delta T_{gr} = \frac{40.308}{cf_1^2}\mathrm{TEC}_{gr}, \tag{1}$$

$$\Delta T_{ph} = -\frac{40.308}{cf_1^2}\mathrm{TEC}_{ph}. \tag{2}$$

Here TEC_{gr} and TEC_{ph} are total electron content along the NS-RS line obtained from group and phase path measurements of the NS signal; ΔT_{gr} and ΔT_{ph} are group and phase ionospheric delay.

$$\mathrm{TEC}_{gr} = \frac{1}{40.308}\frac{f_1^2 f_2^2}{f_1^2 - f_2^2}(P_1 - P_2 + \delta P), \tag{3}$$

$$\mathrm{TEC}_{ph} = \frac{1}{40.308}\frac{f_1^2 f_2^2}{f_1^2 - f_2^2}(L_1\lambda_1 - L_2\lambda_2 + \delta L\lambda + K_n), \tag{4}$$

where P_1 and P_2 are the group paths of the NS radio signal; L_1 and L_2 stand for the number of phase rotations of the NS radio signal obtained during its propagation along the NS-RS path at two coherently coupled frequencies f_1=1575.42 MHz and f_2=1227.6 MHz; $L_1\lambda_1$ and $L_2\lambda_2$ are the phase paths of the NS radio signal; δP and $\delta L\lambda$ are the instrument errors in determining the group and phase ranges to the NS, and $K_n = (N_{f_1}\lambda_1 - N_{f_2}\lambda_2)$, where N_{f_1} and N_{f_2} are the unknown initial whole numbers of phase rotations (phase ambiguity) at frequencies f_1 and f_2 between NS and RS.

Note that the accuracy of reconstruction of TEC values is not affected by such components of the telemetry error as tropospheric delay, clock signatures, errors introduced by selective availability, and other components whose influence on the accuracy of distance measurements is identical in both GPS working frequencies [*Blewitt*, 1990; *Marel and Georgiadou*, 1994].

In updating Klobuchar's model, we used ΔT values obtained from phase distance measurements, which ensure a more accurate reconstruction of TEC values as compared with group measurements. The phase ambiguity was eliminated by adjusting phase measurements to group measurements during their combined processing [*Blewitt*, 1990; *Banyai*, 1997].

The basic expression for calculating ΔT value in Klobuchar's model algorithm is of the form

$$\Delta T^* = \begin{cases} F_n DC & , \quad 20 < T < 8 \\ F_n\left[DC + A_m\cos\left(\dfrac{2\pi(T-T_p)}{P}\right)\right] & , \quad 8 < T < 20 \end{cases} \tag{5}$$

where F_n is the inclination factor, taking into account the NS elevation to the horizon; ΔT^* is the predicted of value ΔT; $P = \beta_0 + \beta_1\Phi_m + \beta_2\Phi_m^2 + \beta_3\Phi_m^3$ is the period of ΔT^* diurnal variation; $A_m = \alpha_0 + \alpha_1\Phi_m + \alpha_2\Phi_m^2 + \alpha_3\Phi_m^3$ is the amplitude of ΔT^* diurnal variation; α_n and β_n are the polynomial coefficients transmitted in block 1 of the information message (ionospheric delay model parameters); Φ_m is the geomagnetic latitude of the subionospheric point; DC = 5 ns is the value of the ΔT^* during the nighttime; and T_p = 1400 LT.

The coefficients α_n and β_n are determined at the main reference station of the ground-based GPS control system using the multiparameter global ionospheric model developed by R. B. Bent describing seasonal ionospheric variations and the average solar emission flux for a current period of time. The values of α_n and β_n determined in this way are used continuously during 5-10 days (depending on heliogeophysical conditions) and are updated thereafter [*Klobuchar*, 1987].

Klobuchar's model is updated from current measurements by fitting T_P and DC values and the parameters α_n and β_n by the criterion of the least sum of squares of deviations of model values of ID (ΔT^*) from measured values (ΔT)

$$U = \sum_{i=1}^{N}(\Delta T_i^* - \Delta T_i)^2 = \min, \tag{6}$$

where N is quantity of measurements; and i is number of a reference in a series of measurements.

To determine the values of model parameters minimizing the functional (6), we used the numerical algorithm (hereinafter referred to as the updating algorithm), based on the combination of the method of rough random search and a discrete analog of the Newton method complemented by the procedure of selecting an optimal step of iteration [*Ermakov and Kalitkin*, 1981]. Such an improvement makes it possible to expand the region of convergence of this method and to improve stability of the solution obtained.

Through the simultaneous use of both methods in terms of a single algorithm, it is possible to avoid the convergence of the computational process to one of the possible local minima in the domain of values of the functional U. In this case, a sufficiently exact solution of this optimization problem can be obtained with low expenditure of computer time and with low requirements in the accuracy of specifying the initial approximate parameters of the ID model. The computational process of the updating algorithm is realized in two stages. In the first stage the method of rough random search is used to determine the values of ionospheric delay model parameters α_n^0, β_n^0, T_p^0, and DC^0, which are then used as zeroth-order approximations in the subsequent computational process from the analog of the Newton method.

3. Quality Evaluation of the Suggested Technique

To assess the quality of the proposed technique for updating Klobuchar's model, predicted values obtained before and after updating the model were reconciled with data of current measurements. In doing so, the value of the standard deviation (SD) of predicted ID values from measured ($\sigma_{\Delta R_i}$) was used as the criterion. The criterion of a minimum ionospheric rms error in determining the user's location was used to evaluate the practical effectiveness of the technique. The proposed technique of updating Klobuchar's model was tested against different series of ionospheric delay measurements obtained in different seasons, times of day, and under different heliogeophysical conditions. To confirm the reliability and quality of results obtained by using the technique of updating Klobuchar's model from current data, examples of several cases are given in Table 1.

In the first case, the measurements were made under conditions of the magnetic storm on February 18, 1999 (K index = [4–7]), during 0200-0400 UT (maximum phase of the magnetic storm) and from 1100 to 1300 UT (recovery storm).

In the second case, a solar eclipse occured on March 9, 1997, in a quiet background geomagnetic situation (Dst = [-3–5], $F_{10.7} = 75$). The measurements were carried out for two periods of time: from 0030 to 0230 UT (totality phase

Table 1. Some Results of Updating the *Klobuchar*'s [1987] Model

	March 9, 1997, UT		Feb. 18, 1999, UT		Dec. 26, 1998, UT	
	0030–0230	1500–1700	0200–0400	1100–1300	0300–0500	1400–1600
T_{upd}, UT	0030–0050	1500–1520	0200–0220	1100–1120	0300–0320	1400–1420
$\sigma_{\Delta R_i,1}$, m	2.62–3.46	0.40–2.66	1.74–4.57	2.65–6.7	1.13–3.61	0.47–2.57
$\sigma_{\Delta R_i,2}$, m	0.89–1.26	0.23–1.00	0.61–1.87	0.51–3.66	0.45–1.89	0.38–1.91
$\sigma_{\mathrm{UPL},1}$, m	9.82	8.15	11.52	19.1	8.76	6.38
$\sigma_{\mathrm{UPL},2}$, m	4.17	4.03	6.87	7.4	4.86	4.90

of eclipse) and from 1500 to 1700 UT (background).

In the third case, the measurements were made on the day with quiet geomagnetic conditions at a moderate level of solar activity (Dst = [-7- -29], $F_{10.7} = 81$), on December 26, 1998. The measurements were also carried out for two periods of time: from 0300 to 0500 UT (daytime) and from 1400 to 1600 UT (nighttime).

Also, in all cases the measurements were made for the same number of navigation satellites (seven) .

Results obtained for the cases under consideration are summarized in Table 1, where the column headings indicate the date of the measurements and the time of day when the measurements were made.

Series of measurements of 20-min duration (T_{upd} in the first row of Table 1) were used in updating the model in all cases. The entire observing period in all the cases under consideration was 2 hours long. The second and third rows in Table 1 include the values of the SD (in meters) between predicted and measured values of ΔT, which were obtained by using Klobuchar's model without updating from current measurements ($\sigma_{\Delta R_i,1}$) and with updating ($\sigma_{\Delta R_i,2}$), respectively. Finally, the fourth and fifth rows of Table 1 contain information about ionospheric rms error in determining the user's location by applying Klobuchar's model without updating ($\sigma_{\mathrm{UPL},1}$) and with updating ($\sigma_{\mathrm{UPL},2}$).

In determining the value of σ_{UPL} in each of the cases under consideration, of the entire number of NS where telemetry measurements were made, we used four NS, for which the least value of the spatial geometric factor was secured. The value of σ_{UPL} was determined as follows:

$$\sigma_{\mathrm{UPL}} = PDOP\left(\sum_{i=1}^{M}\sigma^2_{\Delta R_i}\right)^{0.5}, \qquad (7)$$

where M is the number of NS used in determining the UPL; $PDOP$ is the spatial geometrical factor; and $\sigma_{\Delta R_i}$ is the ionospheric error of determination of the phase range to the i-NS.

We determine the value of $\sigma_{\Delta R_i}$ as the standard deviation between the predicted and measured value of the ionospheric delay

$$\sigma_{\Delta R_i} = \left(\frac{1}{N}\sum_{j=1}^{N}(c\Delta T^j_{ph,i} - c\Delta T^*_{i,j})^2\right)^{0.5}, \qquad (8)$$

where N is the number of measurements; $c\Delta T^j_{ph,i}$ is an additional increment in the phase path of the radio signal from the i-NS, arising as the signal propagates through the ionosphere at the j-point in time; and $c\Delta T^*_{i,j}$ is the predicted of value $c\Delta T^j_{ph,i}$.

In the cases under consideration, the proposed updating technique permitted us to reduce the SD between predicted and measured ΔT values by a factor of 1.2–2.2 for the magnetic storm conditions, by a factor of 2.4–2.8 in the case of the solar eclipse, and by a factor of 1.3–2.0 for the undisturbed ionosphere.

In this case, the ionospheric rms error in determining the user's location for the examples reported here were reduced by factors of 1.6–2.6, 2.0–2.3, and 1.3–1.8, respectively.

4. Conclusions

Using the technique of updating the global ID model, it is possible to reduce the departure of predicted values of ΔT from measured values at a given point of space under particular conditions.

Predictions of the ionospheric delay using the original Klobuchar model under magnetic storm conditions provides a higher accuracy during the main phase, but it decreases considerably for the recovery period (Table 1) which follows the main phase and sets in on the same day (in the example under consideration). Therefore it is more appropriate to update the ionospheric delay model under disturbed ionospheric conditions.

In the nighttime, as follows from (5), Klobuchar's model can be updated only by fitting the DC parameter. Therefore the prediction accuracy of ΔT values is only slightly improved by using the proposed technique for the nighttime.

The implementation of the technique of updating Klobuchar's model also makes it possible to achieve a reduction of the ionospheric rms error in determining the user's location; therefore this technique can be used to reduce the error of coordinate-time determinations in GPS.

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